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TECHNICAL NOTE 3313

SOME MEASUREMENTS OF ATMOSPHERIC TURBULENCE
OBTAINED FROM FLOW-DIRECTION VANES
MOUNTED ON AN AIRPLANE

By Robert G. Chilton

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SUMMARY

The power spectrum of relatively short wavelength turbulence in the atmosphere was calculated from measurements made in flight. The range of wavelengths covered by these measurements was from 10 feet to 200 feet. The power spectral density varies with the square of the wavelength of the turbulence. This variation is in agreement with the high-frequency asymptote of the spectrum form generally assumed for isotropic turbulence. Flow-direction vanes were used to measure the vertical and horizontal components of gust velocity normal to the flight direction. The power spectral densities of the two components are, for practical purposes, equal. The use of vanes is shown to afford a simple, direct method of obtaining the power spectral density of atmospheric turbulence in the relatively high frequency range of airplane response.

INTRODUCTION

The introduction of the methods of generalized harmonic analysis to measurements of atmospheric turbulence has provided a form of statistical information which makes it possible to analyze problems concerning the motion or the flight path of an airplane which is flying in rough air. Because of the random nature of turbulence, previous methods available to airplane designers for describing rough air provided only statistical estimates of peak values of gust velocity, velocity gradients, and the frequency of their occurrence. Although this information is useful for calculating maximum values of airplane gust response, it does not describe the gust disturbance in sufficient detail for calculating the average effect of the continuous gust disturbance on the response of an airplane which is encountering atmospheric turbulence. The airplane response to gusts is determined by the dynamic characteristics of the airplane. The response is, therefore, dependent upon the gust wavelength. Calculating the response of an airplane in

turbulent air requires information pertaining to the gust disturbance which distinguishes between gusts of different wavelengths. The application of the methods of generalized harmonic analysis to gust data accomplishes this purpose.

The function which gives the information in the desired form is called the power spectral density. The power spectral density of gust vertical velocity, for example, is a function of the inverse wavelength which gives a measure of the mean-square velocity of all those components of vertical velocity having wavelengths within an incremental band width. The mean-square gust velocity is, therefore, equal to the area under the power-spectral-density curve. The mean-square value of a function has by electrical analogy come to be considered in terms of average power. The power spectral density of a function, therefore, describes the manner in which the total average power of the function is distributed over the entire frequency range. A complete mathematical development of the concepts of the power spectral density of a random function can be found in a number of texts (for example, ref. 1). Some particular applications of the power-spectrum concept to airplane response to atmospheric turbulence can be found in references 2 and 3. Some of the problems involved in the calculation of the power spectrum from a finite time series are treated in reference 4.

The usefulness of the power-spectrum concept of random functions lies in the theorem that the power spectrum of the response of a linear system subjected to a random disturbance is equal to the power spectrum of the disturbance multiplied by the amplitude squared of the frequency-response function of the system. It is this relationship which makes possible the evaluation of the effect of atmospheric turbulence on airplanes and airplane control systems. Some examples of problems concerning atmospheric turbulence which can be analyzed in this manner are airplane passenger comfort, tracking or bombing errors due to flight-path response of airplanes, and the effects of the combined spanwise and chordwise variations of angle of attack on a wing in turbulent air.

Before these methods can come into general usage, the designer must have available reliable information in the form of spectral densities of atmospheric turbulence. The need exists, therefore, for extensive measurements of turbulence over a wide range of wavelengths representing many different meteorological and topographical conditions. With sufficient data, it can be determined to what extent the nature of turbulent air can be generalized or what special conditions must be imposed before a generalized power spectrum can be applied to specific problems. The first work in this direction (ref. 2) presents data and describes a method of obtaining the power spectrum of gust velocities in the frequency range normally called the short-period range of an airplane. For the airspeed of the airplane in reference 2, this short-period range corresponds to a range of gust wavelengths from approximately

200 feet to 2,000 feet. Other work has been done to extend measurements to very low frequencies or long wavelengths.

The purpose of this paper is to present the spectrum of some short-wavelength measurements of gust velocities. The range of wavelengths covered is approximately from 200 feet to 10 feet. The data presented, when considered with measurements at the lower frequencies, are useful in extending to higher frequencies the data available on which approximations of gust power spectral densities can be based. The measurements were obtained by the use of flow-direction vanes mounted on a boom ahead of the nose of an airplane. This method provides a ready means for accumulating a large amount of data in this frequency range and it can be employed simultaneously with other methods for making measurements in the lower frequency ranges.

SYMBOLS

$G(f)$	power spectral density
f	frequency, cycles per second or cycles per foot
T	finite interval of time, sec; also, scale of turbulence, ft
w	vertical velocity, ft/sec
t	time, sec
l	distance from angle-of-attack vane to center of gravity of airplane, ft
$j = \sqrt{-1}$	
X	longitudinal axis of reference fixed in airplane
Z	normal axis of reference fixed in airplane
θ	airplane angle of pitch, radians
V	airplane forward velocity, ft/sec
α	angle of attack, radians
[]	frequency-response-function notation

Subscripts:

i	input
o	output
g	gust
v	vane-indicated
a	airplane

A dot over a quantity indicates differentiation with respect to time.

INSTRUMENTATION AND TESTS

Flow-direction vanes on a fighter airplane were used to record angle of attack and angle of sideslip (fig. 1). The vanes were mass balanced and located 54 inches ahead of the nose of the airplane. The undamped natural frequency of the vanes at sea level and at a test airspeed of 220 feet per second was approximately 45 cycles per second. The damping ratio was 60 percent of critical. These values were obtained by wind-tunnel tests. The recording system was properly damped and had an undamped natural frequency of approximately 100 cycles per second. Standard NACA airspeed-altimeter, linear-acceleration, angular-velocity, and attitude-pitch recorders were used. A statoscope having a sensitive differential pressure cell with a scale range of ± 3 inches of water was employed to measure variations in pressure altitude.

The airplane was flown in a straight path at an altitude of approximately 1,000 feet with a minimum of control application for periods of 1 minute or less. Test runs were made with the airplane flying both parallel and perpendicular to the wind direction over commercial and light-industrial areas in the vicinity of Portsmouth, Va. The test runs were made in clear air on two afternoons in late summer. The dates and other information pertinent to the test runs are given in table I.

METHOD OF ANALYSIS

Definitions

The relationships which pertain to the concept of the power spectral density of a random function are presented here as defined in reference 1.

Gust vertical velocity is chosen as the function for which the spectral density is of interest. If $w_g(t)$ is defined as the vertical component of gust velocity, the spectral density of w_g can be defined in the following manner. For $-T \leq t \leq T$, let

$$f(t) = w_g(t) \quad (1a)$$

and elsewhere,

$$f(t) = 0 \quad (1b)$$

The Fourier transform of $f(t)$ exists and is by definition

$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-2\pi jft}dt = \int_{-T}^T f(t)e^{-2\pi jft}dt \quad (2)$$

The power spectral density of gust vertical velocity is then defined as

$$G(f) = \lim_{T \rightarrow \infty} \frac{1}{T} |F(f)|^2 \quad (3)$$

for which only positive frequencies are considered. The mean-square value of gust vertical velocity $\overline{w_g^2}$ is found by integrating the power spectrum

$$\overline{w_g^2} = \int_0^{\infty} G(f)df \quad (4)$$

The output spectrum $G_o(f)$ of the response of an airplane (which is assumed to be a linear system) in rough air which has the spectrum $G_1(f)$ is (ref. 1, p. 288)

$$G_o(f) = |Y(2\pi jf)|^2 G_1(f) \quad (5)$$

where $Y(2\pi jf)$ is the frequency-response function which relates the response of the airplane to the gust disturbance.

Measuring Technique

Most of the methods of obtaining the power spectrum of atmospheric turbulence which have been reported require the use of the frequency-response function which relates the response of the airplane to the gust input. In utilizing this technique, the response of the airplane in rough air is measured and the power spectral density of the response is computed. The power spectrum of the gust-velocity input is then calculated by using equation (5). When possible, the response is measured with the controls fixed. If the length of record required is too great for the airplane to stay in trim without control corrections, the effects of the control inputs must be removed and the analysis becomes cumbersome. In this event, a stabilized airplane may be used, in which case it is necessary to know the frequency-response function of the airplane-autopilot combination.

Since the measurements described in this paper are confined to high-frequency gusts (1 cycle per second and higher at an airspeed of 220 feet per second), it is possible to employ a flow-direction vane as the primary means of measurement. The gust velocity can be calculated from the indications of the vane and corrected for the effect of the airplane response to the gusts. No frequency-response functions for the airplane are required, and no restrictions on the application of controls are necessary.

Geometrical Relationships

The geometrical relationships for the airplane and flow-direction or angle-of-attack vane are shown in figure 2. All angles are assumed to be small variations from the steady state, and small angle values are assumed for the trigonometric functions. If variations in upwash are neglected, the vane-indicated angle of attack α_v can be expressed as

$$\alpha_v = \alpha_a + \alpha_g - \frac{l\dot{\theta}}{V} \quad (6)$$

where l is the distance from the vane to the center of gravity of the airplane. The term $l\dot{\theta}$ is the vertical velocity at the vane resulting from pitching velocity of the airplane. The boom is assumed to be rigid.

Since $\alpha_a = \theta + \frac{w_a}{V}$ and $\alpha_g = \frac{w_g}{V}$, the gust velocity w_g can be written

as

$$w_g = V(\alpha_v - \theta) - w_a + l\dot{\theta} \quad (7)$$

The only quantity not measured directly is the airplane vertical velocity which can be calculated by integrating the normal accelerations as discussed in the following section.

Method of Calculation

The quantity (gust vertical velocity) for which the power spectral density is required is given in equation (7) as the sum of measured or calculated quantities. The primary quantity is the vane measurement and those describing airplane reactions are correction terms. The simplest way of handling this problem is to perform first the arithmetical calculations indicated in equation (7) to obtain a single record of computed gust vertical velocities. The results can then be processed to obtain the power spectral density of the gusts. The only quantity not measured directly is the vertical velocity of the airplane. Normal acceleration and variations in pressure altitude were measured to provide information for calculating the airplane vertical velocity. The normal accelerations were integrated and the slope of the pressure-altitude curve provided the initial value of vertical velocity. The calculated vertical velocities were integrated a second time for comparison with the pressure-altitude curve.

Since gusts of relatively short wavelength were of primary interest, the assumption that the angle-of-attack vane gives a sufficiently accurate measurement of the gust vertical velocity without corrections ($w_g = V\alpha_v$) was considered. This assumption would provide a great saving in the film reading required to correct for the airplane response. For this purpose a trial record of approximately 20 seconds duration was analyzed to determine the importance of the corrections. The power spectral density of the gust vertical velocity calculated from equation (7) is compared in figure 3 with that obtained by assuming no airplane response. The comparison indicates that, above 1 cycle per second, it is unnecessary to correct the vane measurements for the effects of the airplane response to the gusts for the particular airplane used. The vertical-velocity response of the test airplane in the range of frequencies of interest is so small that the corrections may be neglected. The frequency range for which this effect is true for other airplanes depends, of course, upon the dynamic characteristics of the particular airplane.

Analysis of Gust-Response Characteristics of

Vane-Airplane Combination

In order to obtain theoretical verification of the apparent fact that the airplane response may be neglected in the calculation of gust

power in the relatively short wavelength range, an analysis of the problem was made in terms of the theoretical frequency-response functions of the airplane. If the assumption is made that there are no applied controls or disturbances other than the gust, the following relationships for the rigid airplane can be written:

$$\left. \begin{aligned} \alpha_a &= \left[\frac{\alpha_a}{\alpha_g} \right] \alpha_g \\ \dot{\theta} &= \left[\frac{\dot{\theta}}{\alpha_g} \right] \alpha_g \end{aligned} \right\} \quad (8)$$

where the bracketed terms represent the frequency-response functions of the airplane which relate the angle of attack and rate of pitch to gust inputs. The performance of the vane as a gust-measuring instrument can be obtained by substituting equations (8) into equation (6) and solving for the ratio of the vane response to the gust input

$$\frac{\alpha_v}{\alpha_g} = 1 + \left[\frac{\alpha_a}{\alpha_g} \right] - \frac{\lambda}{V} \left[\frac{\dot{\theta}}{\alpha_g} \right] \equiv \left[\frac{\alpha_v}{\alpha_g} \right] \quad (9)$$

The airplane-frequency-response functions which appear in equation (9) were calculated in an unpublished analysis of the two-degree-of-freedom response of the test airplane for the test conditions. The analysis included the effects of unsteady lift on the pitching moment and lift due to gusts as well as an approximation of the effect of a nonuniform distribution of angle of attack across the wing span due to short-wavelength gusts. With these airplane frequency-response functions, the frequency-response function of the vane-airplane combination as an instrument for measuring gust vertical velocity was calculated from equation (9) and plotted in figure 4. From this figure it appears that, for the test configuration of vane and airplane, the vane performs very well as a gust-measuring instrument at frequencies above 1 cycle per second. Below 1 cycle per second it would seem that the vane measurements would be useful, if corrected for the effect of the airplane response, down to approximately 0.1 cycle per second. Below this frequency the airplane response becomes the greater part and vane measurements are of no further use for obtaining gust vertical velocity.

The fractional error in the power spectral density of vane-indicated gust vertical velocity without corrections for the airplane response can be determined in terms of the performance function of equation (9) by the use of the theorem of equation (5) as follows:

$$\frac{\text{PSD}(w_g) - \text{PSD}(w_v)}{\text{PSD}(w_g)} = 1 - \frac{\text{PSD}(w_v)}{\text{PSD}(w_g)} = 1 - \left[\frac{\alpha_v}{\alpha_g} \right]^2 \quad (10)$$

where $\text{PSD}()$ represents the power spectral density of the quantity in parentheses and w_v is the vane-indicated vertical velocity ($w_v = V\alpha_v$). The error in the vane-measured power spectrum due to airplane response can be calculated from figure 4 and equation (10). The error is less than 10 percent for frequencies greater than 1 cycle per second.

PRESENTATION OF RESULTS

The power spectral density of gust vertical velocity was calculated from measurements made with an angle-of-attack vane by following the procedure recommended in reference 4. Figure 5 represents the average power spectrum obtained from three test runs of from 50 to 60 seconds each made on two different days. The data approximate a straight line which has a slope of -2 when plotted on logarithmic paper. Each point has associated with it a band of confidence from the standpoint of statistical reliability within which the true value lies for a given probability (refs. 3 and 4). The confidence band for 90-percent probability was calculated for the faired spectrum and is shown by the dashed lines in figure 5. Each calculated point represents the average power contained in the band width which is centered about it and extends halfway to the adjacent points. It should be noticed that the band width over which the average is taken is increased with increasing frequency with the result that the reliability is greater at the higher frequencies.

The band widths over which the average power was calculated are illustrated better in figure 6. In this figure are compared the power spectra of gust vertical velocity calculated from an upwind test run and a crosswind test run which were obtained on the same day only a few minutes apart. The results are plotted as bar graphs. The height of each bar or step represents the average power density for the band width over which it extends. The 90-percent confidence limits for figure 6 are the same as those given in figure 7, which shows a comparison of the power spectra of gust vertical velocity and gust horizontal velocity normal to the flight path recorded simultaneously. The 90-percent confidence limits, although shown related to the straight-line fairing of the vertical-velocity spectrum, are applicable to both. The assumption of no response of the airplane is made in the calculation of each spectrum.

The standard deviations of gust vertical velocity for the upwind and crosswind test runs on the same day were 2.9 and 3.0 feet per second, respectively. The standard deviation for the vertical and horizontal

components of gust velocity recorded simultaneously were 2.1 and 4.8 feet per second, respectively. The comparatively high standard deviation of the horizontal velocity is a result of low-frequency power in the record which resulted from a lightly controlled Dutch roll oscillation of the airplane and is noticeable in the low-frequency values of figure 7.

The probability distribution calculated for the 20-second trial record used in obtaining figure 3 is shown in figure 8 for comparison with the fitted normal distribution.

DISCUSSION

The power spectrum of gust vertical velocity is presented in figure 5. For comparison, the lower frequency spectrum of reference 2 is also shown. Both spectra are compared with a straight line which has a slope of -2. It is evident that atmospheric turbulence is well represented by a function which varies inversely with the square of the frequency. The spectrum form which has been found to be suitable for describing isotropic turbulence for measurements obtained in this manner is represented by the function (ref. 5)

$$G(f) = G(0) \frac{1 + 3(2\pi fT)^2}{[1 + (2\pi fT)^2]^2} \quad (11)$$

The constant T is the scale of turbulence which has the dimensions of seconds or feet depending upon whether f is frequency or inverse wavelength. The constant $G(0)$ is proportional to both the scale of turbulence and the mean-square turbulence velocity. The high-frequency asymptote of equation (11) is proportional to f^{-2} which is in agreement with the experimental observations. The theoretical spectrum has a break frequency at the value for which the wavelength is equal to $2\pi T$. Below this frequency the value of the power spectrum approaches the constant value $G(0)$. None of the data of figure 5 extend to wavelengths long enough to confirm this characteristic.

The power spectrum of figure 5 is the average of three records, two of which were taken on the same day only a few minutes apart. One of the two records which were taken on the same day was taken while flying upwind; the other was taken while flying crosswind. In figure 6 the power spectral densities of these two records are compared. The average wind direction appears to have no effect on the measurements. The spectra in figure 6 are presented as bar graphs in order to illustrate better the band widths over which the average power is calculated. The same band widths are used in figures 5 to 7.

The third record was obtained on a different day having similar weather conditions. This record included simultaneous measurements of angle of attack and angle of sideslip. The power spectral densities of gust vertical velocity and gust horizontal velocity normal to the flight path are compared in figure 7. The calculations for each component were made by assuming no airplane response. The statistical confidence limits are shown for the faired vertical-velocity spectrum. Similar limits are applicable to the horizontal component. With allowances for the large low-frequency power in the horizontal-velocity spectrum which is the result of a lightly controlled Dutch roll oscillation of the airplane, the agreement between the two components is good.

Many problems concerned with gust disturbances involve considerations of normality, homogeneity, and isotropy of turbulent air. The probability distribution of gust vertical velocity was calculated only for the 20-second trial record which was used to determine the effect of neglecting the airplane response. Figure 8 shows the result compared with the fitted normal distribution. Figures 6 to 8 may encourage assumptions with regard to considerations of normality and isotropy. Figure 7 suggests that at least two-dimensional isotropy exists. The use of angle-of-attack and angle-of-sideslip vanes in this fashion with the addition of a sensitive airspeed recording system for measuring airspeed fluctuations due to gusts would be very useful for making detailed studies of isotropy of atmospheric turbulence. By measuring three components of gust velocity with respect to the airplane, theoretical relationships between longitudinal and transverse correlations for isotropic turbulence could be checked.

The results of this and other experimental determinations of the spectrum of atmospheric turbulence seem to justify the use of the spectrum form of equation (11) for those problems that require the determination of the effect of rough air on the performance of an aircraft control system. More long-wavelength data are needed in order to define the spectrum in the vicinity of the break frequency. The two primary characteristics of the turbulence spectrum are the scale and intensity. It must be determined over what range of values these characteristics vary and, from the standpoint of statistical probability, what design values should be chosen for specific problems. In particular, the value of the scale of turbulence is important in those problems for which the frequency range of interest is affected by the break frequency of the turbulence spectrum.

For the purpose of accumulating data on atmospheric turbulence over a wide range of wavelengths, flow-direction vanes of the type used in this investigation should be very useful except at the very low frequencies with respect to the response of airplanes. As was indicated in figure 4 and the section entitled "Method of Analysis," the angle-of-attack vane gives a direct measurement of the gust vertical velocity at

those frequencies for which the airplane response is negligible. At lower frequencies the vane indications can be corrected for the response of the airplane to afford a measurement of the gust vertical velocity. In this range of frequencies the gust power spectrum which is calculated from the corrected vane indications can be used to calculate the amplitude characteristics of the airplane frequency-response function or to check the derived frequency-response function. At still lower frequencies, the airplane response quantities are the primary measurement; so the frequency-response function for the airplane-autopilot combination must be used (since it seems desirable to stabilize the phugoid mode of the airplane under these conditions).

CONCLUDING REMARKS

A flow-direction vane mounted on an airplane has been shown to provide a good measurement of gust velocity in the range of frequencies above the short-period response of the airplane. This use of vanes affords a simple, direct method of obtaining the power spectral density of atmospheric turbulence in that frequency range. By correcting the vane measurements for airplane response to gusts, the spectrum can be extended to lower frequencies which include a large part of the short-period range. Therefore, when airplane response is measured for the purpose of calculating the power spectrum of gusts, an advantage is to be gained by recording angle-of-attack and angle-of-sideslip vane indications simultaneously. In addition to extending the range over which the spectrum can be calculated to higher frequencies, the spectrum calculated from the vane readings, when corrected for airplane response, can be used to check the results of the lower frequency measurements which require the use of theoretical airplane-response characteristics.

The measured power spectral density of gusts displays an inverse-square relationship with frequency. That is, the average power per unit band width varies with the square of the wavelength over the frequency range covered. The wavelengths corresponding to the frequency range covered in these measurements are from 220 feet to 10 feet. This inverse-square variation follows the trend exhibited in other measurements of gust spectra covering lower frequency bands. Until there are accumulated sufficient data on the power spectral density of atmospheric turbulence to set up design criteria based on statistical considerations, this form of the spectrum of turbulence should be useful for investigating the effects of turbulence over those frequency ranges of interest to airplane or control-system designers, provided the scale of turbulence can be assumed to be large. For those problems for which the frequency range of interest is affected by the break frequency of the turbulence spectrum, a value of scale must be assumed for use with the theoretical spectrum.

The power spectral densities of gust vertical velocity and gust horizontal velocity perpendicular to the flight path were found to be, for practical purposes, equal. This result is suggestive of two-dimensional isotropy. Information concerning isotropy in atmospheric turbulence in the high-frequency range could be obtained readily by use of a sensitive airspeed recording system with angle-of-attack and angle-of-sideslip vanes for measuring three components of gust velocity with respect to the airplane.

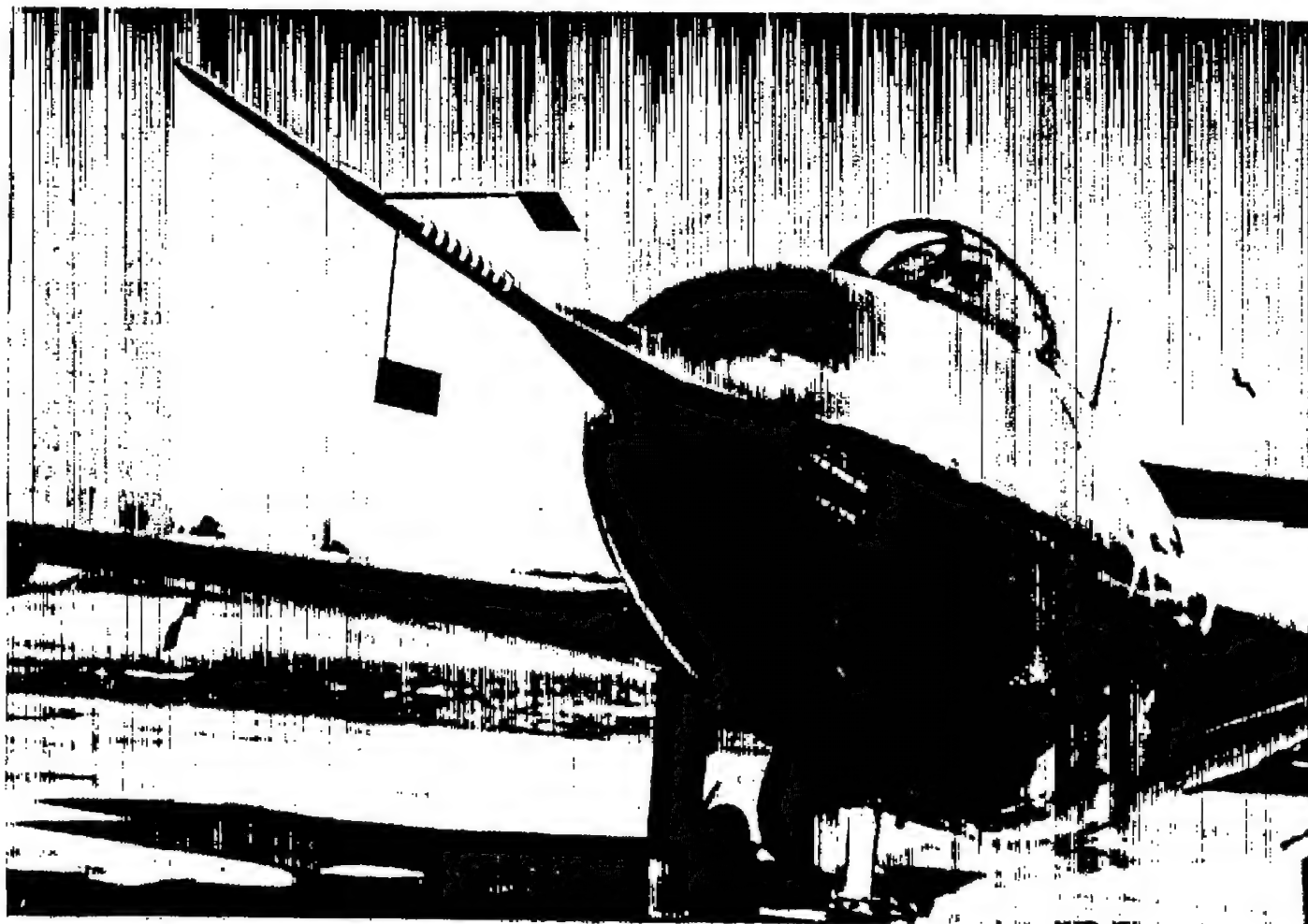
Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 24, 1954.

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TABLE I.- SUMMARY OF OBSERVED METEOROLOGICAL AND TEST CONDITIONS

	Test		
	1, Aug. 22, 1952	2, Aug. 22, 1952	3, Sept. 9, 1952
Record length, sec	56	52	60
Altitude, ft	1,000	900	600
Free-air temperature, °C	27	27	22
Wind direction	330	330	50
Approximate wind velocity, knots . . .	8	8	16
Flight direction	Upwind	Crosswind	---
Standard deviation of vane-indicated gust vertical velocity, ft/sec . . .	2.9	3.0	2.1
Standard deviation of vane-indicated gust horizontal velocity, ft/sec . .	-----	-----	4.8



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Figure 1.- Installation of flow-direction vanes for measuring gust velocities.

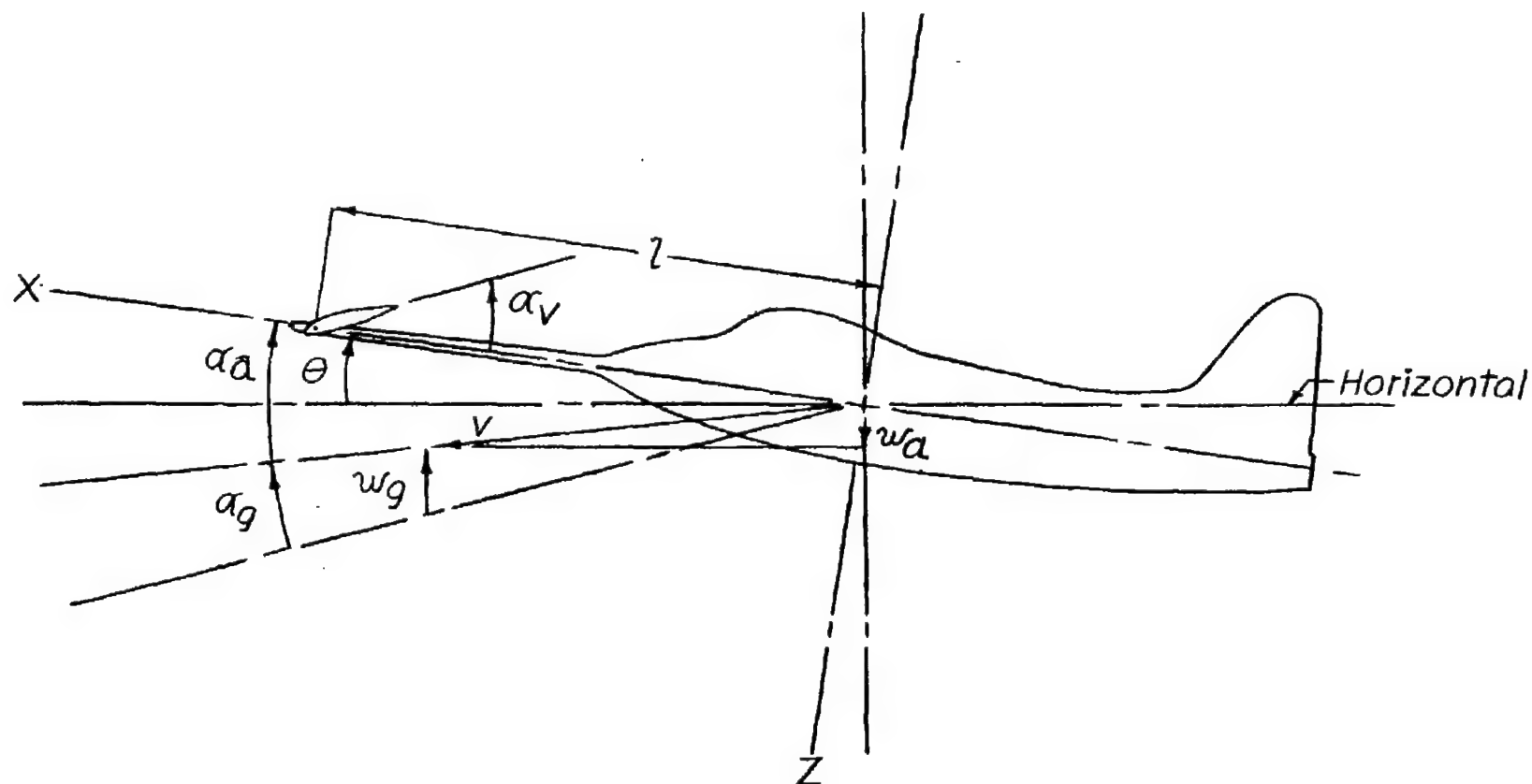


Figure 2.- Symbols and axes used in analysis. Positive direction of angles and velocities is shown.

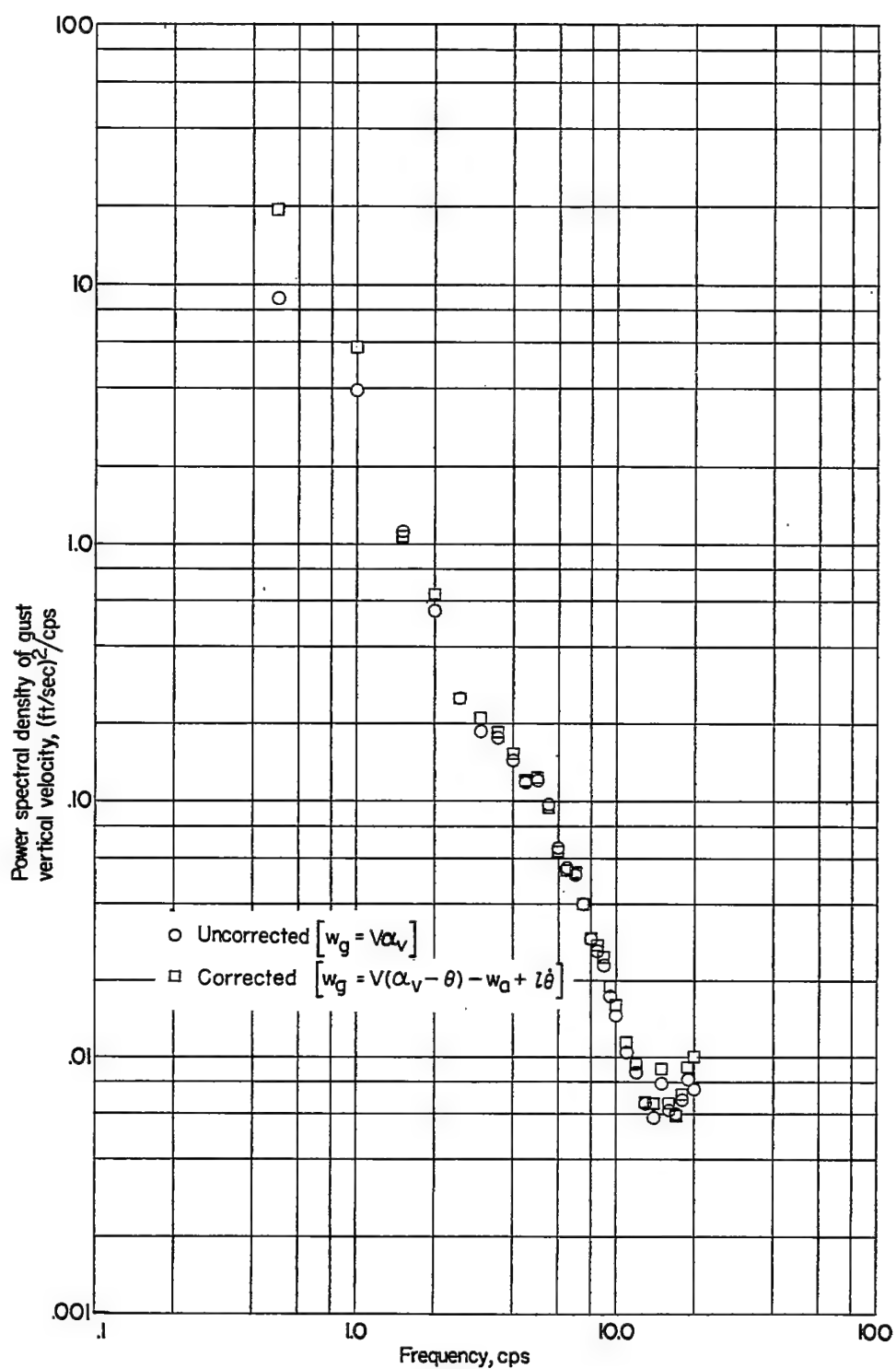


Figure 3.- Comparison of power spectral density of gust vertical velocity as obtained from corrected and uncorrected flow-direction vane. $V = 220$ feet per second.

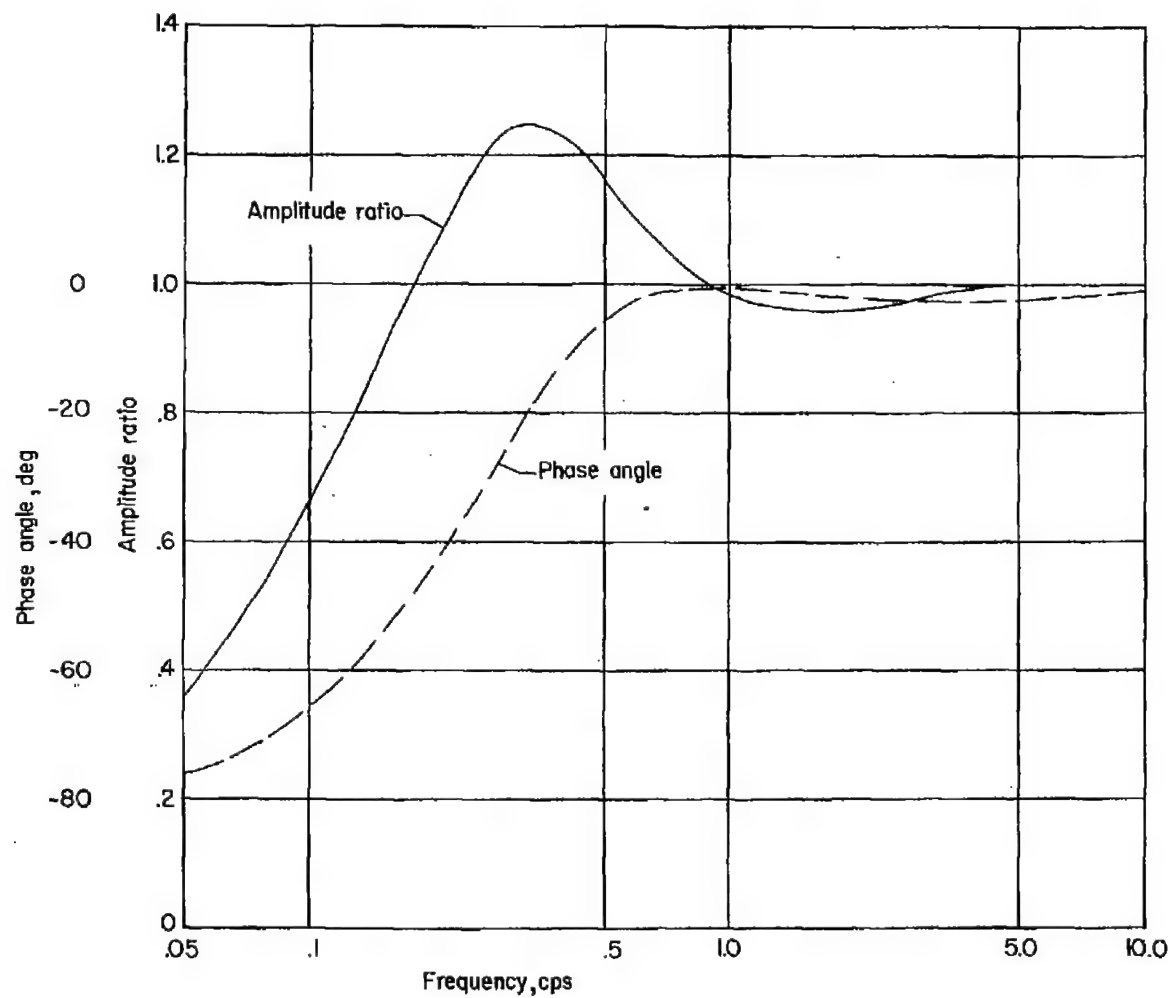


Figure 4.- Frequency-response function which describes the performance of vane-airplane combination as gust-measuring instrument, $\left[\frac{a_v}{a_g} \right]$.
 $V = 220$ feet per second.

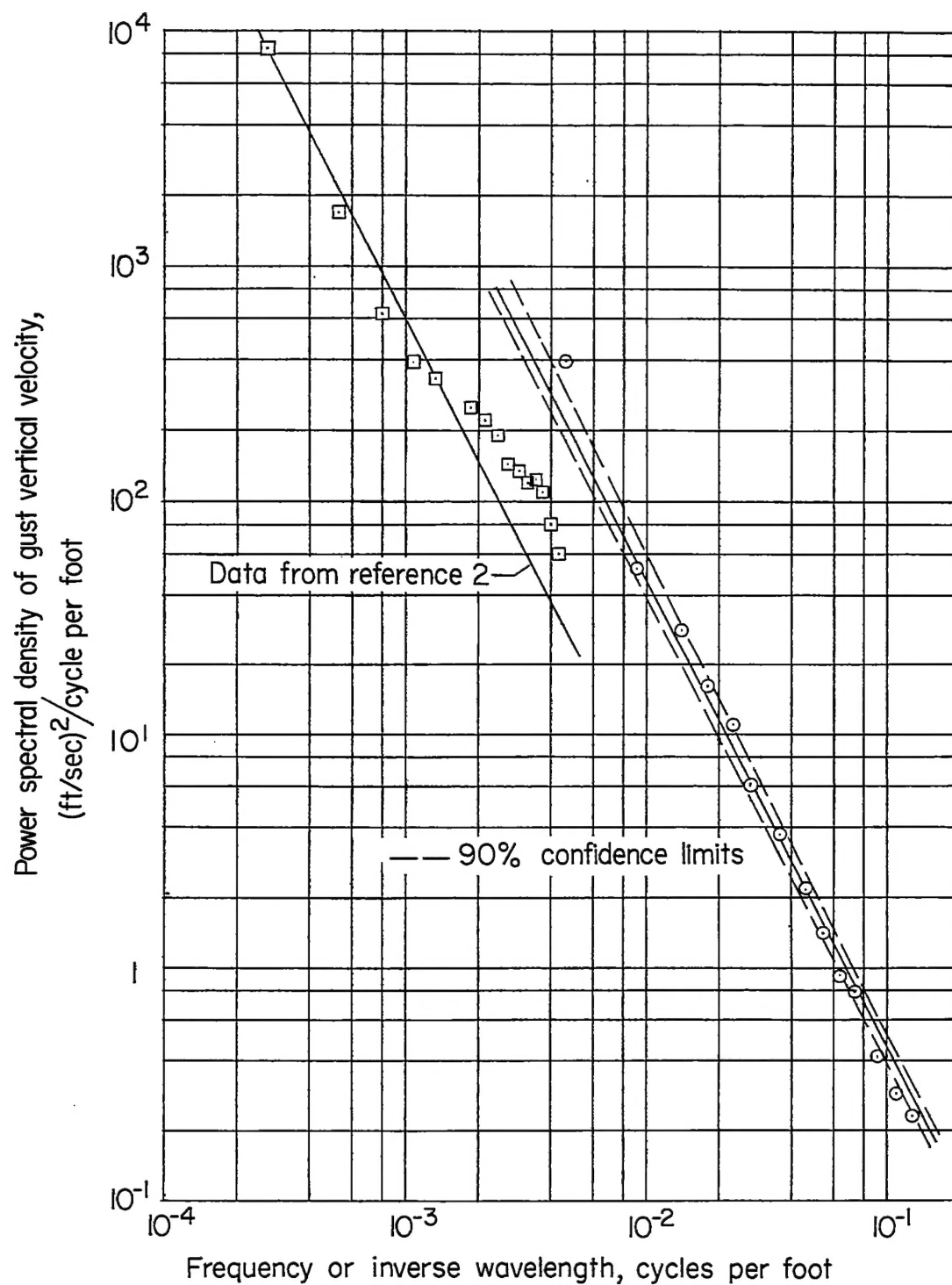


Figure 5.- Average power spectrum of gust vertical velocity obtained from three records shown for comparison with data of reference 2.

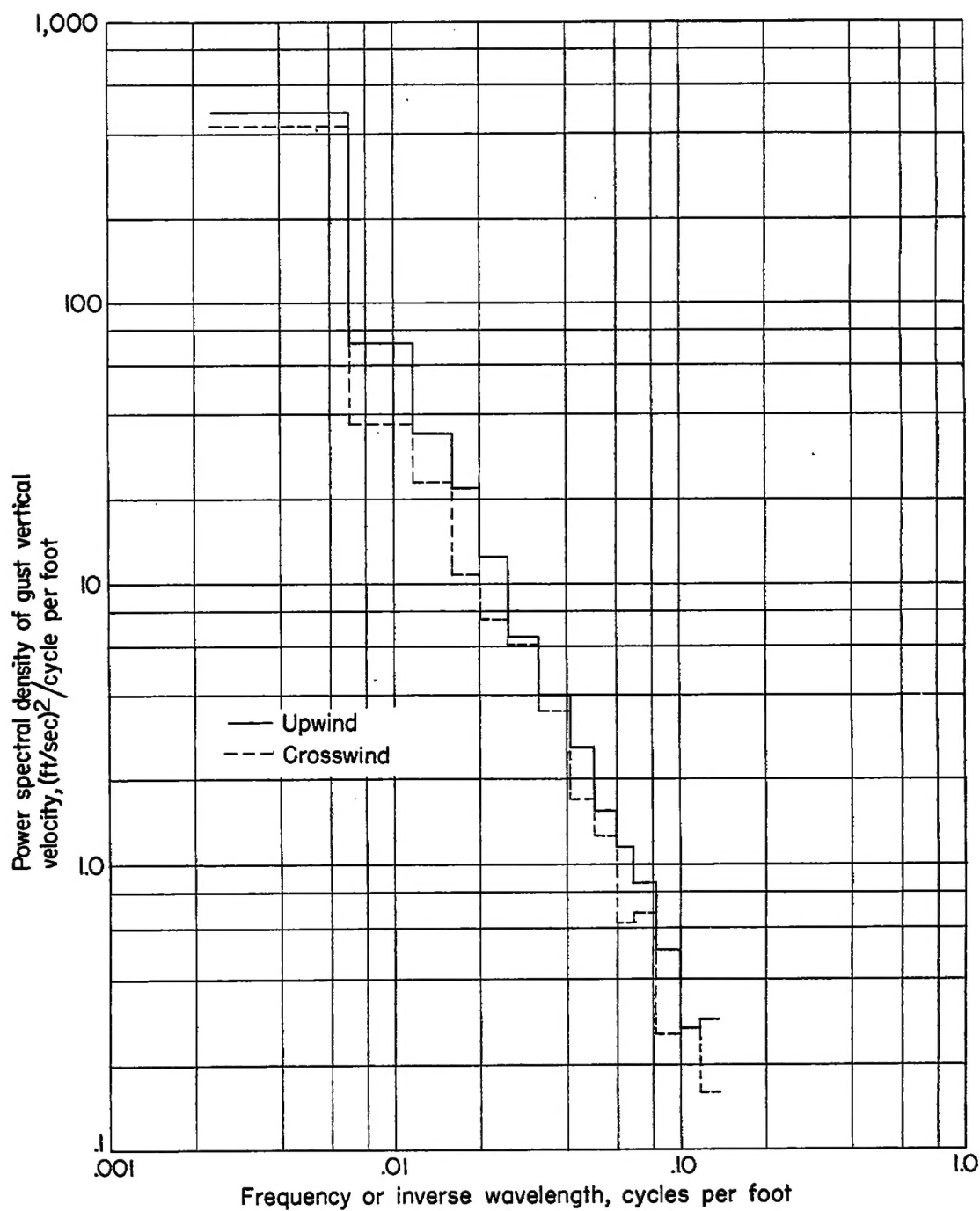


Figure 6.- Comparison of power spectral density of gust vertical velocity calculated from an upwind test run and a crosswind test run in the same area. $V = 220$ feet per second.

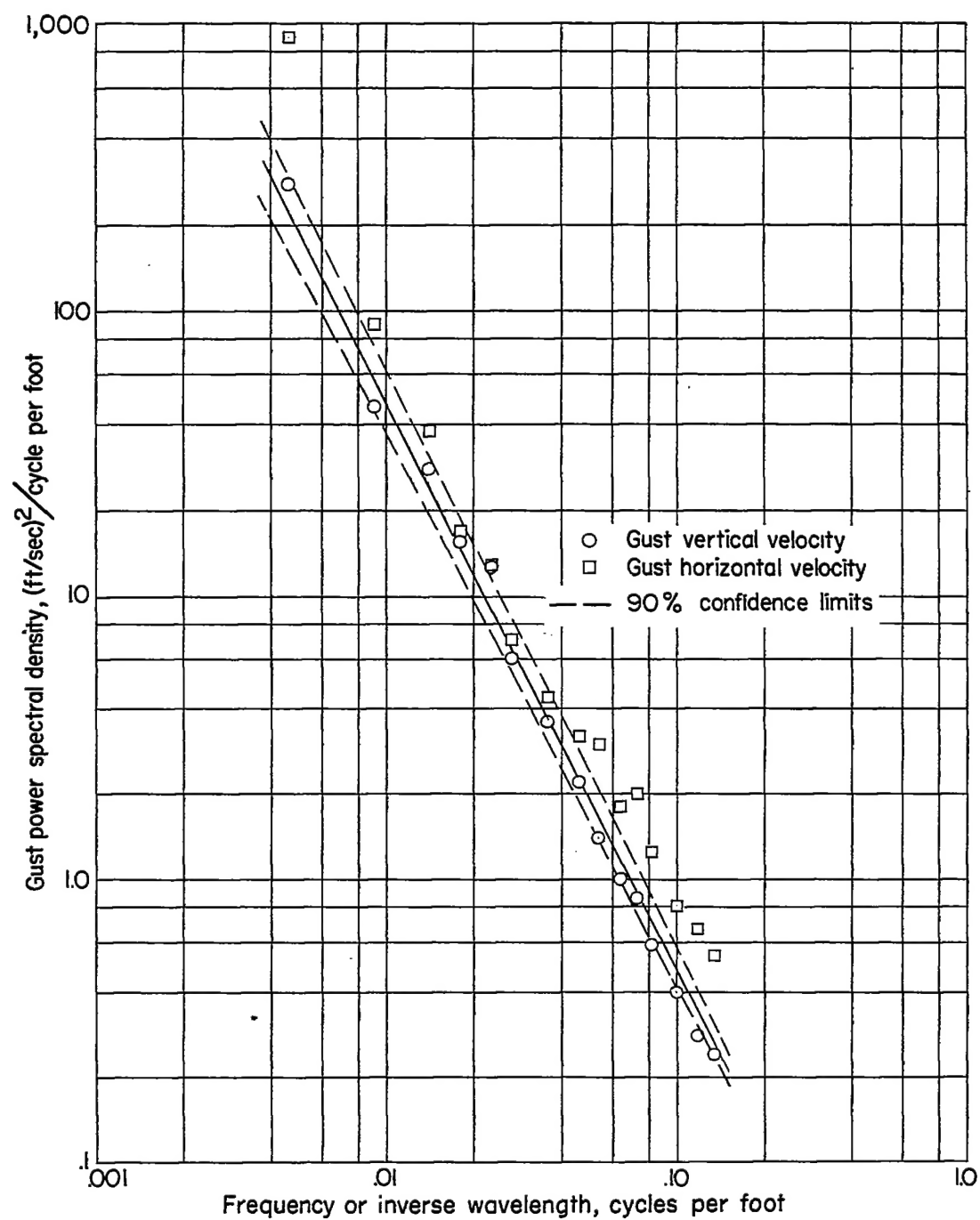


Figure 7.- Comparison of power spectral density of gust vertical velocity and horizontal velocity normal to flight path recorded simultaneously. $V = 220$ feet per second.

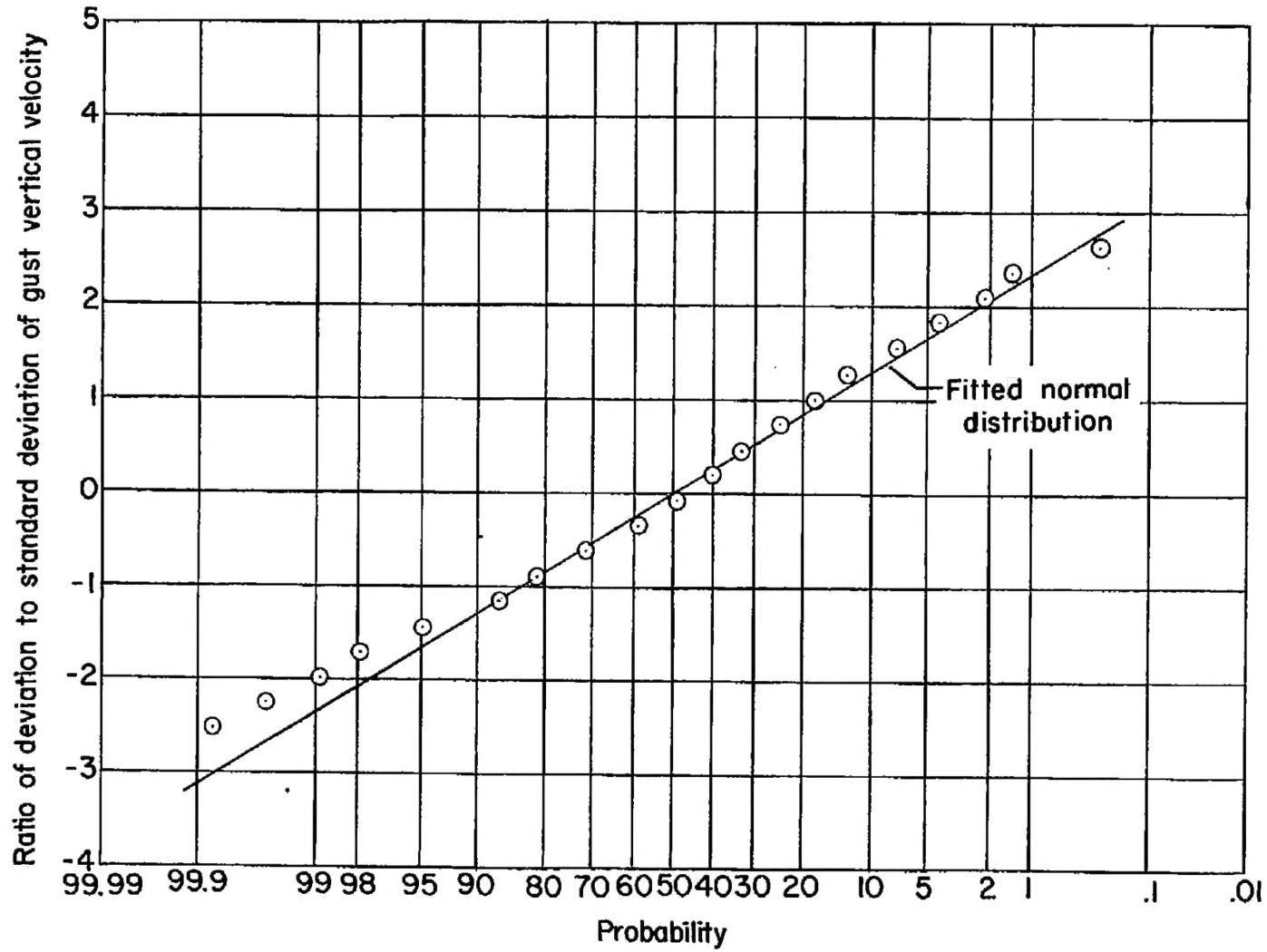


Figure 8.- Probability that ratio of deviation to standard deviation of gust vertical velocity will exceed a given value.